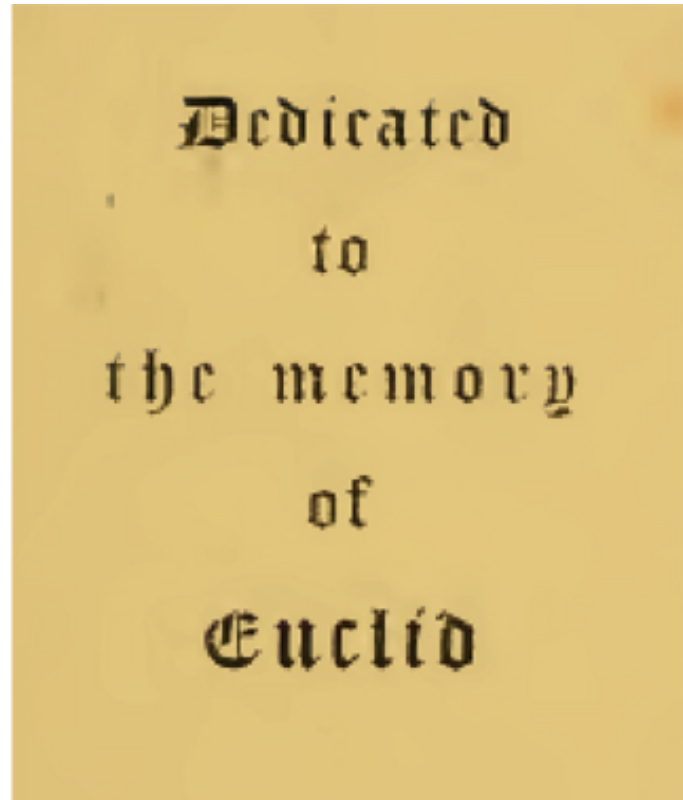
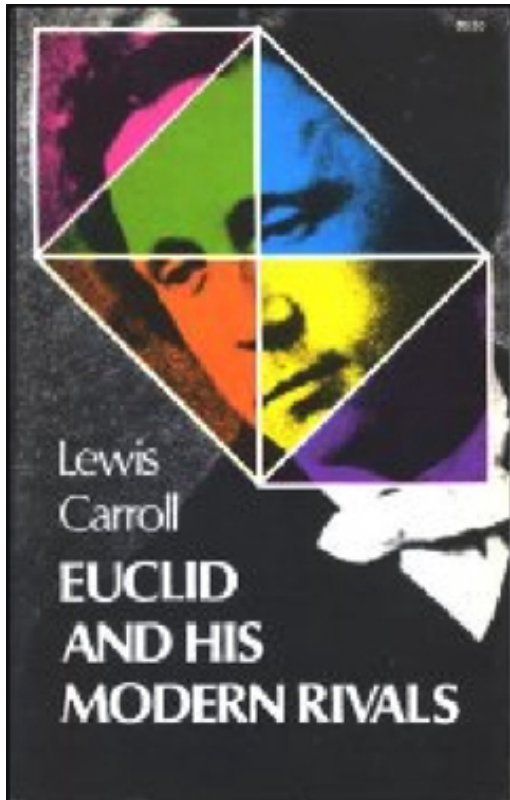


### 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---



***Euclid and His Modern Rivals*** (1879) es una comedia en cuatro actos –el trabajo más famoso de Lewis Carroll en geometría–, una defensa de la geometría de Euclides frente a *teorías matemáticas modernas*

Se trata del análisis meticuloso de varios libros destinados a la enseñanza de la geometría elemental en las escuelas de aquella época. En estos manuales –y respecto al libro de *Los Elementos* de Euclides– se modifica a veces algún axioma o una definición, en otros se cambia el orden de los teoremas, en ocasiones se abordan las demostraciones de manera diferente, en algunos se modifica el tratamiento de la teoría de las paralelas...

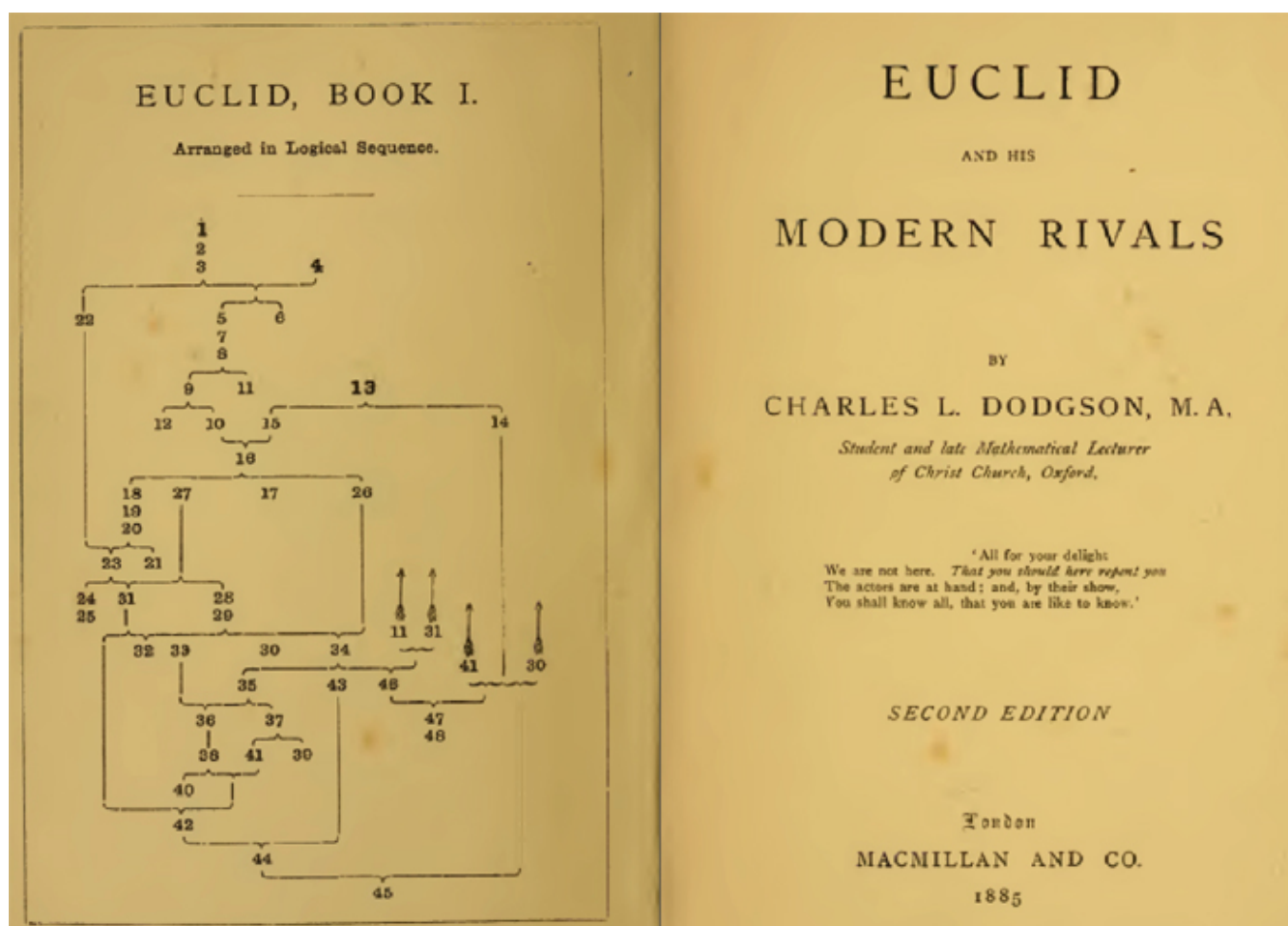
Para realizar este análisis –que podría resultar tedioso de otra manera, como afirma el propio autor en la introducción– Carroll recurre a [Minos](#) y [Radamantis](#) –dos de los tres jueces de [Hades](#) –, árbitros estrictos que dialogan con dos fantasmas: el de Euclides –modesto, y aunque

### 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

convencido de la calidad de su obra, no tiene inconveniente en que se analice— y el del profesor alemán Herr Niemand, portavoz de los 13 autores cuyos libros se examinan. Uno a uno, escena a escena, estos *rivales modernos* verán como sus textos se critican y se rechazan frente al manual de Euclides.

Lewis Carroll se sirve del humor y de los juegos de palabras para invalidar a los rivales de Euclides.



El texto finaliza con el discurso de despedida de Euclides, tras el cual los fantasmas desaparecen y Minos se va a dormir:

### 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---

*'The cock doth crow, the day doth daw', and all respectable ghosts ought to be going home. Let me carry with me the hope that I have convinced you of the importance, if not the necessity, of retaining my order and numbering, and my method of treating straight Lines, angles, right angles, and (most especially) Parallels. Leave me these untouched, and I shall look on with great contentment while other changes are made while my proofs are abridged and improved, while alternative proofs are appended to mine and while new Problems and Theorems are interpolated.*

*In all these matters my Manual is capable of almost unlimited improvement.*

Como anécdota, el primer logotipo de Wikipedia [\[1\]](#) —conocido como *Wiki logo Nupedia*— se diseñó en 2001, superponiendo una frase de Lewis Carroll sobre un círculo, usando el efecto de ojo de pez para simular una esfera. La frase es una cita en inglés tomada del prefacio de ***Euclid and his Modern Rivals***, que dice:



*In one respect this book is an experiment, and may chance to prove a failure: I mean that I have not thought it necessary to maintain throughout the gravity of style which scientific writers usually affect, and which has somehow come to be regarded as an 'inseparable accident' of scientific teaching. I never could quite see the reasonableness of this immemorial law: subjects there are, no doubt, which are in their essence too serious to admit of any lightness of treatment – but I cannot recognise Geometry as one of them. Nevertheless it will, I trust, be found that I have permitted myself a glimpse of the comic side of things only at fitting seasons, when the tired reader might well crave a moment's breathing-space, and not on any occasion where it could endanger the continuity of the line of argument.*

### 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---

Para concluir, incluyo la exhaustiva descripción –actos y escenas con detalle de lo que sucede en cada una de ellas– del argumento de la obra tal y como se incluye en el texto de Lewis Carroll.

#### **ACTO I: *Preliminaries to examination of Modern Rivals***

##### **ESCENA I** [Minos y Radamantis]

Consequences of allowing the use of various Manuals of Geometry : that we must accept

- (1) 'Circular'arguments
- (2) Illogical do.

Example from Cooley

Example from Wilson

##### **ESCENA II.** [Minos y Euclides]

##### **§ 1: *A priori reasons for retaining Euclid's Manual***

We require, in a Manual, a selection rather than a complete repertory of Geometrical truths

Discussion limited to subject-matter of Euc. I, II

One fixed logical sequence essential

One system of numbering desirable

*A priori* claims of Euclid's sequence and numeration to be retained

New theorems might be interpolated without change of numeration

##### **§ 2: *Method of procedure in examining Modern Rivals***

Proposed changes which, even if proved to be essential, *would not* necessitate the

### 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---

abandonment of -Euclid's Manual:

- (1) Propositions to be omitted;
- (2) Propositions to be replaced by new proofs;
- (3) New propositions to be added.

Proposed changes which, if proved to be essential, would necessitate such abandonment:

- (1) Separation of Problems and Theorems;
- (2) Different treatment of Parallels.

Other subjects of enquiry:

- (3) Superposition;
- (4) Use of diagonals in Euc. II;
- (5) Treatment of Lines;
- (6) Treatment of Angles;
- (7) Euclid's propositions omitted;
- (8) Euclid's propositions newly treated;
- (9) New propositions ;
- (10) Style, &c.

List of authors to be examined, viz.:

Legendre, Cooley, Cuthbertson, Henrici, Wilson, Pierce, Willock, Chauvenet, Loomis, Morell, Reynolds, Wright, Syllabus of Association -for Improvement of Geometrical Teaching, Wilson's 'Syllabus'-Manual.

#### **§ 3: *The combination, or separation, of Problems and Theorems***

Reasons assigned for separation

Reasons for combination:

- (1) Problems are also Theorems;
- (2) Separation would necessitate a new numeration,
- (3) and hypothetical constructions.

#### **§ 4: *Syllabus of propositions relating to Pairs of Lines***

Three classes of Pairs of Lines:

- (1) Having two common points;

### 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---

- (2) Having a common point and a separate point;
- (3) Having no common point.

Four kinds of 'properties';

- (1) common or separate points;
- (2) equality, or otherwise, of angles made with transversals;
- (3) equidistance, or otherwise, of points on the one from the other;
- (4) direction.

Conventions as to language

Propositions divisible into two classes:

- (1) Deducible from undisputed Axioms;
- (2) Deducible from disputable Axioms.

Three classes of Pairs of Lines:

- (1) Coincidental;
- (2) Intersectional;
- (3) Separational.

Subjects and predicates of Propositions concerning these three classes:

Coincidental

Intersectional

Separational

**TABLE I.** *Containing twenty Propositions, of which some are undisputed Axioms, and the rest real and valid Theorems, deducible from undisputed Axioms*

Subjects and predicates of other propositions concerning Separational Lines

**TABLE II.** *Containing eighteen Propositions, of which no one is an undisputed Axiom, but all are real and valid Theorems, which, though not deducible from undisputed Axioms, are such that, if any one be admitted as an Axiom, the rest can be proved*

**TABLE III.** *Containing five Propositions, taken from Table II, which have been proposed as Axioms:*

- (1) Euclid's Axiom;
- (2) T. Simpson's Axiom;
- (3) Clavius' Axiom;
- (4) Playfair's Axiom;
- (5) R. Simpson's Axiom.

It will be shown (in Appendix III) that *any* Theorem of Table II is sufficient logical basis for all the

### 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---

rest

#### § 5: *Playfair's Axiom*

Is Euclid's 12 Axiom axiomatic?

Need of test for meeting of finite Lines

Euclid's and Playfair's Axioms deducible, each from the other

Reasons for preferring Euclid's Axiom:

- (1) Playfair's does not show *which way* the Lines will meet;
- (2) Playfair's asserts more than Euclid's, the additional matter being superfluous

Objection to Euclid's Axiom (that it is the converse of I. 17) untenable

#### § 6: *Principle of Superposition*

Used by Moderns in Euc. I. 5

Used by Moderns in Euc. I. 24

#### § 7: *Omission of Diagonals in Euc. II*

Proposal tested by comparing Euc. II. 4, with Mr. Wilson's version of it

**ACTO II:** [Minos y Niemand] *Manuals which reject Euclid's treatment of Parallels*

**ESCENA I:** *Introductory*

**ESCENA II:** *Treatment of Parallels by methods involving infinite series*

### 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---

#### LEGENDRE

Treatment of Line  
Treatment of Angle  
Treatment of Parallels  
Test for meeting of finite Lines  
Manual unsuited for beginners

#### **ESCENA III: *Treatment of Parallels by angles made with transversals***

##### COOLEY

Style of Preface  
Treatment of Parallels  
Utter collapse of Manual

#### **ESCENA IV: *Treatment of Parallels by equidistances***

##### CUTHBERTSON

Treatment of Line  
Attempted proof of Euclid's (tacitly assumed) Axiom, that two Lines cannot have a common segment  
Treatment of Angle  
Treatment of Parallels  
Assumption of R. Simpson's Axiom  
Euclid's 12th Axiom replaced by a Definition, two Axioms, and five Theorems  
Test for meeting of two finite Lines  
Manual a modified Euclid

#### **ESCENA V: *Treatment of Parallels by revolving Lines***

##### HENRICH

Treatment of Line  
Treatment of Angle  
Treatment of Parallels  
Attempted proof of Playfair's Axiom discussed  
Attempted proof of Playfair's Axiom rejected  
General survey of book:

Enormous amount of new matter



### 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---

Two 'non-sequitur'  
An absurdity proved *à la Henrici*  
Motion 'per saltum' denied  
A strange hypothesis  
A new kind of 'open question'  
Another 'non-sequitur'  
An awkward corner  
Theorems on Symmetry  
Summary of faults  
Euclid I, 18, 19, contrasted with Henrici  
A final tit-bit

Manual rejected

#### **ESCENA VI: *Treatment of Parallels by direction***

##### **§1: WILSON**

Introductory  
Treatment of Line  
Treatment of Angle  
Extension of limit of 'angle' to sum of four right angles  
'Straight' angles  
Meaning of 'direction'  
'Opposite' directions  
'Same' and 'different' directions  
Axiom 'different Lines may have the same direction' discussed  
Property 'same direction', when asserted of different Lines, can neither be defined, nor constructed, nor tested  
'Separational directions' *not* identical with 'identical directions'  
Virtual assumption of 'separational Lines are real' (which Euclid proves in I. 27), as Axiom  
'different Lines may have the different direction' discussed  
Axiom 'different Lines may have the same direction' rejected, and Axiom 'different Lines may have the different direction' granted with limitations  
Axiom 'different which meet one another have different directions' granted  
Axiom 'Lines with different directions would meet' discussed and rejected  
Diagram of 'same' and 'different' directions condemned  
'Different but with the same direction' accepted as (ideal) definition by Pair of Lines  
'Parallel' as used by Wilson, to be replaced by term 'sepcodal'  
Definition discussed  
Theorem 'sepcodal' related Lines do not meet accepted  
Theorem 'Lines sepcodal related to a third, are so to each other' discussed, and condemned as

### 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---

a '*Petitio Principii*'

Axiom 'Angle may be transferred, preserving directions of sides' discussed

If angle be variable, it involves fallacy '*Adicto secundum Quid ad dictum Simpliciter*'

If it be constant., the resulting Theorem (virtually identical with the Axiom) involves fallacy '*Petitio Principii*'

If angle be constant, the Axiom involves two assumptions: viz. that

- (1) there can be a Pair of different Lines that make equal angles with *any* transversal
- (2) Lines, which make equal angles with a certain transversal, do so with *any* transversal

Axiom rejected

Ideas of 'direction' discussed

Theory of 'direction' unsuited for teaching

Test for meeting of finite Lines discussed:

it virtually involves Euclid's Axiom  
or if not, it causes hiatus in proofs

List of Euclid's propositions which are omitted  
General survey of book:

A false Corollary

A plethora of negatives

A superfluous *datum*

Cumbrous proof of Euc. I. 24

An unintelligible Corollary

A unique 'Theorem of equality'

A bold assumption

Two cases of '*Petitio Principii*'

A problem 3½ pages long >

A fifth case of '*Petitio Principii*'

A sixth

Summing-up, and rejection of Manual.

### §2: PIERCE

Treatment of Line

### 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---

Introduction of Infinitesimals

Treatment of Parallels

Angle viewed as 'difference of direction'

Assumption of Axiom 'different Lines may have same direction'

List of Euclid's Theorems which are omitted

Manual *not* adapted for beginners

#### §3: WILLOCK

Treatment of Parallels

Virtual assumption of Axiom 'different Lines may have the same direction'

Assumption of Axiom 'separational Lines have the same direction'

General survey of book:

Difficulties introduced too soon

Omission of 'coincidental' Lines

'Principle of double conversion' discussed, and condemned as illogical

Mysterious passage about 'incommensurables'

Manual rejected

#### **ACTO III: *Manuals which adopt Euclid's treatment of Parallels***

#### **ESCENA I**

§1: Introductory

## 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---

### § 2: CHAUVENET

General survey

### § 3: LOOMIS

General survey

### § 4: MORELL

Treatment of Line

Treatment of Angle

Treatment of Parallels

General survey:

'Direct', 'reciprocal', and 'contrary' Theorems

Sentient points

A false assertion

A speaking radius

Ratios and common measures

Derivation of 'homologous'

Mensuration of areas 146

A logical *fiasco*

Manual rejected

### 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---

#### § 5: REYNOLDS

General survey

List of Euclid's Theorems omitted

#### § 6: WRIGHT

Quotations from preface

General survey

Specimen of verbose obscurity

ESCENA II

#### § 1: *Syllabus of the Association for the Improvement of Geometrical Teaching*

Introduction of *Nostradamus*, a member of the Association

Treatment of Line

Treatment of Angle

Treatment of Parallels

Test for meeting of finite Lines

Re-arrangement of Euclid's Theorems

General survey:

A 'Theorem' is a 'statement of a Theorem'

Rule of Conversion

Miscellaneous inaccuracies

Summing-up

## 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---

### § 2: *Wilson's 'Syllabus'-Manual*

Introductory

A Theorem is a 'statement of a Theorem'

Rule of Conversion

Every Theorem a 'means of measuring'

'Straight angles'

Miscellaneous inaccuracies

The Manual's one great merit

No test for meeting of finite Lines

Propositions discussed in detail:

An important omission

An illogical conversion

*'Un enfant terrible'*

Summary of results:

Of 73 propositions of Euclid, this Manual has 14 omitted;

43 done as in Euclid;

10 done by new but objectionable methods, viz.

1 illogical;

1 'hypothetical construction';

2 needlessly using 'superposition';

2 algebraical;

4 omitting the diagonals of Euc. II.;

6 done by new and admissible methods.

No reason for abandoning Euclid's sequence and numeration

Nor for regarding this Manual as anything but a revised Euclid

Summing-up

**ACTO IV:** [Minos y Euclides] *Manual de Euclides*

## 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---

### § 1: *Treatment of Pairs of Lines*

Modern treatment of Parallels  
Playfair's Axiom  
Test for meeting of finite Lines

### § 2: *Euclid's constructions*

'Arbitrary restrictions'  
'Exclusion of hypothetical constructions'

### § 3: *Euclid's demonstrations*

PAGE  
'Invariably syllogistic form'  
'Too great length of demonstration'  
'Too great brevity of demonstration'  
'Constant reference to axioms'

### § 4: *Euclid's style*

Artificiality, unsuggestiveness, and want of simplicity

### § 5: *Euclid's treatment of Lines and Angles*

### 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---

Treatment of Line  
Treatment of Angle :

'declination from' accepted  
must be less than sum of two right angles  
'multiple angles' in VI. 33  
proof for Ax. 10 accepted

#### **§ 6: Omissions, alterations, and additions, suggested by Modern Rivals**

Omission of I. 7 suggested  
Reasons for retaining it:

needed to prove I. 8  
not included in new I. 8  
proves *rigidity* of Triangle .  
I. 7, 8 analogous to III. 23, 24  
bears on practical science

Omission of II. 8 suggested

Reason for retaining it, its use in Geometrical Conic Sections

Alterations suggested:

New proofs for I. 5  
'hypothetical construction'  
superposition  
treating sides as 'obliques'  
treating sides as radii of a Circle  
Inversion of order of I. 8, 24; rejected  
Inversion of order of I- 18, 19, 20; do.  
Fuller proof of I. 24 ; accepted  
Algebraical proofs of II. ; rejected

Additions suggested:

New Axiom; accepted



## 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---

Two new Theorems ; do.

### **§7: *The summing-up***

Euclid's farewell speech

## APPENDICES

**I.** Extract from Mr. Todhunter's essay on 'Elementary Geometry' included in 'The Conflict of Studies, &c'

**II.** Extract from Mr. De Morgan's review of Mr. Wilson's Geometry, in the 'Athenoeum' for July 18, 1868

**III.** Proof that, if any one proposition of Table II be granted as an Axiom, the rest can be deduced from it

**IV.** List of propositions of Euc. I, II, with references to their occurrence in the manuals of his Modern Rivals:

**§1.** References to Legendre, Cuthbertson, Henrici, Wilson, Pierce and Willock

**§2.** References to the other Modern Rivals

Nota:

### 63. (Septiembre 2012) Euclides y sus rivales modernos, de Lewis Carroll

Escrito por Marta Macho Stadler (Universidad del País Vasco)  
Miércoles 19 de Septiembre de 2012 16:00

---

[1] En 2000 Jimbo Wales creó **Nupedia**, un proyecto de enciclopedia libre basado en un ambicioso proceso de revisión por pares. Debido al lento avance del proyecto, en 2001 se creó un wiki –UseModWiki– vinculado a Nupedia cuya finalidad inicial era agilizar la creación de artículos de forma paralela, antes de que éstos pasaran al sistema de revisión por expertos. El éxito de aquel proyecto paralelo – **Wikipedia**– acabó eclipsando a **Nupedia**, que dejó de funcionar en 2003.