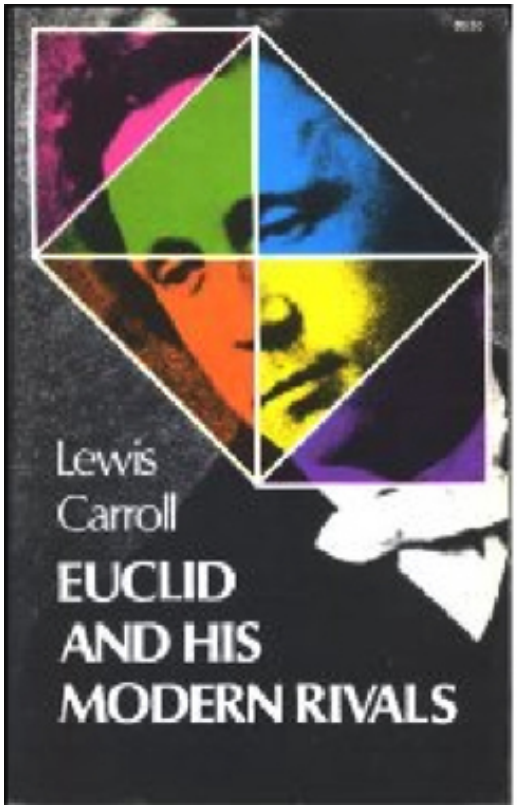
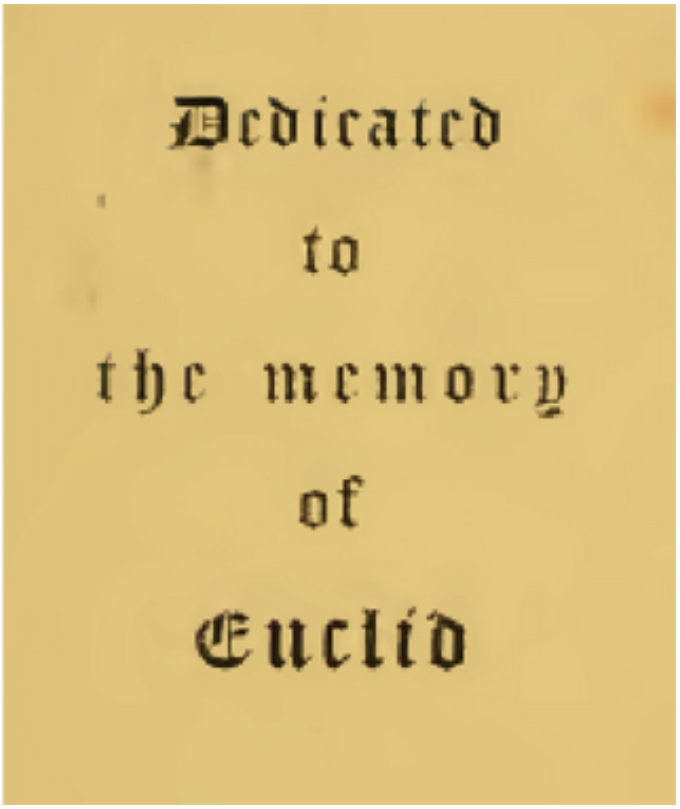
Escrito por Marta Macho Stadler (Universidad del País Vasco)
Miércoles 19 de Septiembre de 2012 16:00





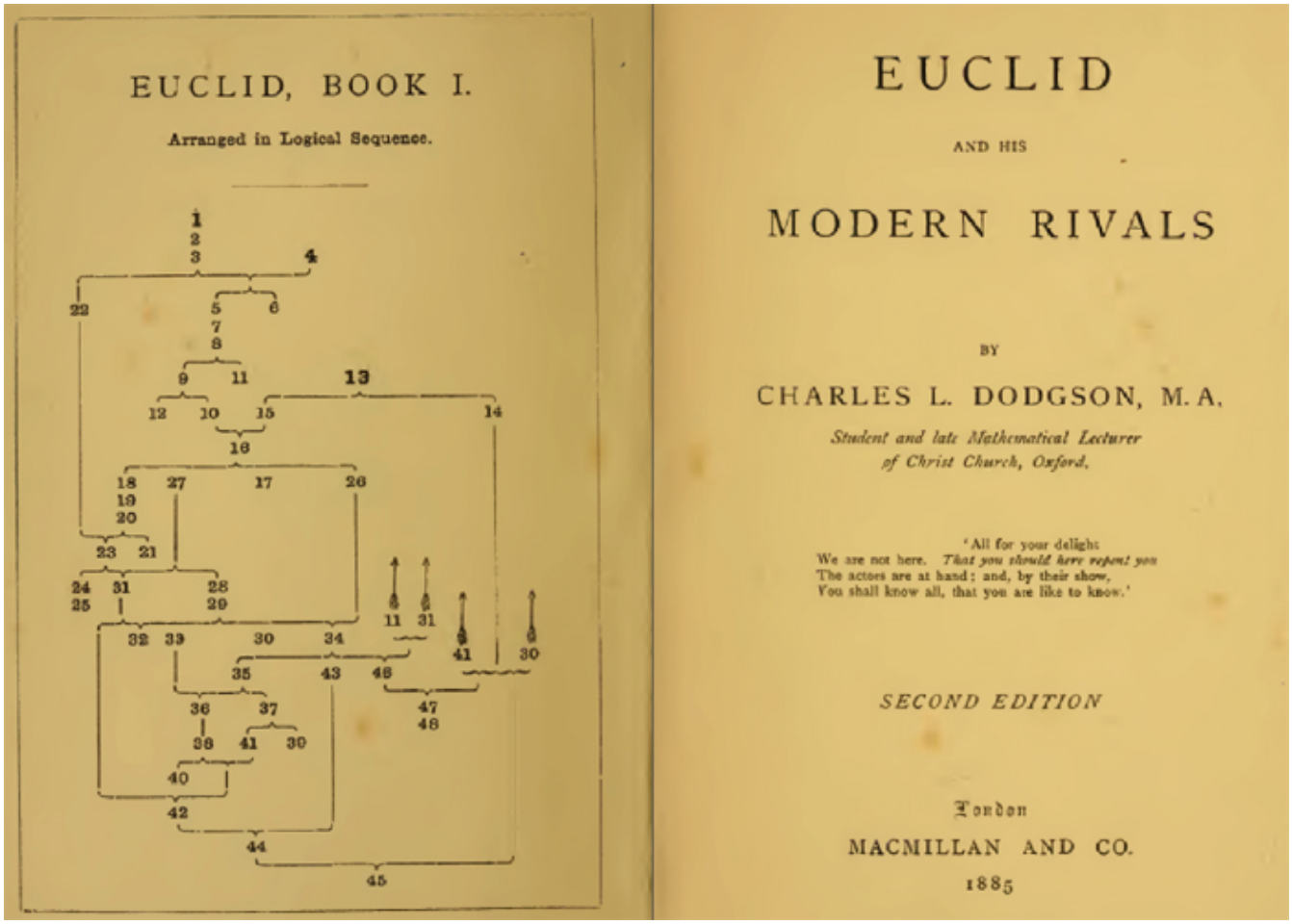
Euclid and His Modern Rivals (1879) es una comedia en cuatro actos –el trabajo más famoso de Lewis Carroll en geometría–, una defensa de la geometría de Euclides frente a *teorías matemáticas modernas*
.

Se trata del análisis meticuloso de varios libros destinados a la enseñanza de la geometría elemental en las escuelas de aquella época. En estos manuales –y respecto al libro de *Los Elementos*
de Euclides– se modifica a veces algún axioma o una definición, en otros se cambia el orden de los teoremas, en ocasiones se abordan las demostraciones de manera diferente, en algunos se modifica el tratamiento de la teoría de las paralelas...

Para realizar este análisis –que podría resultar tedioso de otra manera, como afirma el propio autor en la introducción– Carroll recurre a Minos y Radamantis –dos de los tres jueces de H ades
–, árbitros estrictos que dialogan con dos fantasmas: el de Euclides –modesto, y aunque

convencido de la calidad de su obra, no tiene inconveniente en que se analice– y el del profesor alemán Herr Niemand, portavoz de los 13 autores cuyos libros se examinan. Uno a uno, escena a escena, estos
rivales modernos
verán como sus textos se critican y se rechazan frente al manual de Euclides.

Lewis Carroll se sirve del humor y de los juegos de palabras para invalidar a los rivales de Euclides.


EUCLID, BOOK I.

Arranged in Logical Sequence.

EUCLID

AND HIS

MODERN RIVALS

BY

CHARLES L. DODGSON, M.A.

Student and late Mathematical Lecturer of Christ Church, Oxford.

'All for your delight
We are not here. *That you should here repent you*
The actors are at hand; and, by their show,
You shall know all, that you are like to know.'

SECOND EDITION

London
MACMILLAN AND CO.
1885

El texto finaliza con el discurso de despedida de Euclides, tras el cual los fantasmas desaparecen y Minos se va a dormir:

'The cock doth craw, the day doth daw', and all respectable ghosts ought to be going home. Let me carry with me the hope that I have convinced you of the importance, if not the necessity, of retaining my order and numbering, and my method of treating straight Lines, angles, right angles, and (most especially) Parallels. Leave me these untouched, and I shall look on with great contentment while other changes are made while my proofs are abridged and improved, while alternative proofs are appended to mine and while new Problems and Theorems are interpolated.

In all these matters my Manual is capable of almost unlimited improvement.

Como anécdota, el primer logotipo de Wikipedia [1] –conocido como *Wiki logo Nupedia*– se diseñó en 2001, superponiendo una frase de Lewis Carroll
sobre un círculo, usando el efecto de ojo de pez para simular una esfera. La frase es una cita en inglés tomada del prefacio de
Euclid and his Modern Rivals
, que dice:



In one respect this book is an experiment, and may chance to prove a failure: I mean that I have not thought it necessary to maintain throughout the gravity of style which scientific writers usually affect, and which has somehow come to be regarded as an 'inseparable accident' of scientific teaching. I never could quite see the reasonableness of this immemorial law: subjects there are, no doubt, which are in their essence too serious to admit of any lightness of treatment – but I cannot recognise Geometry as one of them. Nevertheless it will, I trust, be found that I have permitted myself a glimpse of the comic side of things only at fitting seasons, when the tired reader might well crave a moment's breathing-space, and not on any occasion where it could endanger the continuity of the line of argument.

Para concluir, incluyo la exhaustiva descripción –actos y escenas con detalle de lo que sucede en cada una de ellas– del argumento de la obra tal y como se incluye en el texto de Lewis Carroll.

ACTO I: ***Preliminaries to examination of Modern Rivals***

ESCENA I [Minos y Radamantis]
Consequences of allowing the use of various Manuals of Geometry : that we must accept

(1) 'Circular'arguments
(2) Illogical do.

Example from Cooley
Example from Wilson

ESCENA II. [Minos y Euclides]

§ 1: ***A priori reasons for retaining Euclid's Manual***
We require, in a Manual, a selection rather than a complete repertory of Geometrical truths
Discussion limited to subject-matter of Euc. I, II
One fixed logical sequence essential
One system of numbering desirable
A priori claims of Euclid's sequence and numeration to be retained
New theorems might be interpolated without change of numeration

§ 2: ***Method of procedure in examining Modern Rivals***
Proposed changes which, even if proved to be essential, *would not* necessitate the

abandonment of -Euclid's Manual:

(1) Propositions to be omitted;
(2) Propositions to be replaced by new proofs;
(3) New propositions to be added.

Proposed changes which, if proved to be essential, would necessitate such abandonment:

(1) Separation of Problems and Theorems;
(2) Different treatment of Parallels.

Other subjects of enquiry:

(3) Superposition;
(4) Use of diagonals in Euc. II;
(5) Treatment of Lines;
(6) Treatment of Angles;
(7) Euclid's propositions omitted;
(8) Euclid's propositions newly treated;
(9) New propositions ;
(10) Style, &e.

List of authors to be examined, viz.:
Legendre, Cooley, Cuthbertson, Henrici, Wilson, Pierce, Willock, Chauvenet, Loomis, Morell, Reynolds, Wright, Syllabus of Association -for Improvement of Geometrical Teaching, Wilson's 'Syllabus'-Manual.

§ 3: ***The combination, or separation, of Problems and Theorems***
Reasons assigned for separation
Reasons for combination:

(1) Problems are also Theorems;
(2) Separation would necessitate a new numeration,
(3) and hypothetical constructions.

§ 4: ***Syllabus of propositions relating to Pairs of Lines***
Three classes of Pairs of Lines:

(1) Having two common points;

(2) Having a common point and a separate point;
(3) Having no common point.

Four kinds of 'properties';

(1) common or separate points;
(2) equality, or otherwise, of angles made with transversals;
(3) equidistance, or otherwise, of points on the one from the other;
(4) direction.

Conventions as to language
Propositions divisible into two classes:

(1) Deducible from undisputed Axioms;
(2) Deducible from disputable Axioms.

Three classes of Pairs of Lines:

(1) Coincidental;
(2) Intersectional;
(3) Separational.

Subjects and predicates of Propositions concerning these three classes:
Coincidental
Intersectional
Separational
TABLE I. *Containing twenty Propositions, of which some are undisputed Axioms, and the rest real and valid Theorems, deducible from undisputed Axioms*
Subjects and predicates of other propositions concerning Separational Lines
TABLE II. *Containing eighteen Propositions, of which no one is an undisputed Axiom, but all are real and valid Theorems, which, though not deducible from undisputed Axioms, are such that, if any one de admitted as an Axiom, the rest can be proved*
TABLE III. *Containing five Propositions, taken from Table II, which have been proposed as Axioms:*

(1) Euclid's Axiom;
(2) T. Simpson's Axiom;
(3) Clavius' Axiom;
(4) Playfair's Axiom;
(5) R. Simpson's Axiom.

It will be shown (in Appendix III) that *any* Theorem of Table II is sufficient logical basis for all the

rest

§ 5: ***Playfair's Axiom***
Is Euclid's 12 Axiom axiomatic?
Need of test for meeting of finite Lines
Euclid's and Playfair's Axioms deducible, each from the other
Reasons for preferring Euclid's Axiom:

(1) Playfair's does not show *which way* the Lines will meet;
(2) Playfair's asserts more than Euclid's, the additional matter being superfluous

Objection to Euclid's Axiom (that it is the converse of I. 17) untenable

§ 6: ***Principle of Superposition***
Used by Moderns in Euc. I. 5
Used by Moderns in Euc. I. 24

§ 7:.***Omission of Diagonals in Euc. II***
Proposal tested by comparing Euc. II. 4, with Mr.Wilson's version of it

ACTO II: [Minos y Niemand] ***Manuals which reject Euclid's treatment of Parallels***

ESCENA I***: Introductory***

ESCENA II: ***Treatment of Parallels by methods involving infinite series***

LEGENDRE
Treatment of Line
Treatment of Angle
Treatment of Parallels
Test for meeting of finite Lines
Manual unsuited for beginners

ESCENA III: ***Treatment of Parallels ly angles made with transversals***
COOLEY
Style of Preface
Treatment of Parallels
Utter collapse of Manual

ESCENA IV: ***Treatment of Parallels by equidistances***
CUTHBERTSON
Treatment of Line
Attempted proof of Euclid's (tacitly assumed) Axiom, that two Lines cannot have a common segment
Treatment of Angle
Treatment of Parallels
Assumption of R. Simpson's Axiom
Euclid's 12th Axiom replaced by a Definition, two Axioms, and five Theorems
Test for meeting of two finite Lines
Manual a modified Euclid

ESCENA V: ***Treatment of Parallels by revolving Lines***
HENRICH
Treatment of Line
Treatment of Angle
Treatment of Parallels
Attempted proof of Playfair's Axiom discussed
Attempted proof of Playfair's Axiom rejected
General survey of book:

Enormous amount of new matter

Two ‘non-sequitur’
An absurdity proved *à la Henrici*
Motion ‘per saltum’ denied
A strange hypothesis
A new kind of ‘open question’
Another ‘non-sequitur’
An awkard corner
Theorems on Symmetry
Summary of faults
Euclid I, 18, 19, contrasted with Henrici
A final tit-bit

Manual rejected

ESCENA VI: *Treatment of Parallels by direction*

§1: WILSON
Introductory
Treatment of Line
Treatment of Angle
Extension of limit of ‘angle’ to sum of four right angles
‘Straight’ angles
Meaning of ‘direction’
‘Opposite’ directions
‘Same’ and ‘different’ directions
Axiom ‘different Lines may have the same direction’ discussed
Property ‘same direction’, when asserted of different Lines, can neither be defined, nor constructed, nor tested
‘Separational directions’ *not* identical with ‘identical directions '
Virtual assumption of ‘separational Lines are real’ (which Euclid proves in I. 27), as Axiom ‘different Lines may have the different direction’ discussed
Axiom ‘different Lines may have the same direction’ rejected, and Axiom ‘different Lines may have the different direction’ granted with limitations
Axiom ‘different which meet one another have different directions’ granted
Axiom ‘Lines with different directions would meet’ discussed
and rejected
Diagram of ‘same’ and ‘different’ directions condemned
‘Different but with the same direction’ accepted as (ideal) definition by Pair of Lines
‘Parallel’ as used by Wilson, to be replaced by term ‘sepcodal’
Definition discussed
Theorem ‘sepcodal’ related Lines do not meet accepted
Theorem ‘Lines sepcodal related to a third, are so to each other’ discussed, and condemned as

a '*Petitio Principii*'
Axiom 'Angle may be transferred, preserving directions of sides' discussed
If angle be variable, it involves fallacy '*Adicto secmdun Quid ad dictum Simpliciter''*
If it be constant., the resulting Theorem (virtually identical with the Axiom) involves fallacy '*Petiti o Principii'*
If angle be constant, the Axiom involves two assumptions: viz. that

(1) there can be a Pair of different Lines that make equal angles with *any* transversal
(2) Lines, which make equal angles with a certain transversal, do so with *any* transversal

Axiom rejected
Ideas of 'direction' discussed
Theory of 'direction' unsuited for teaching
Test for meeting of finite Lines discussed:

it virtually involves Euclid's Axiom
or if not, it causes hiatus in proofs

List of Euclid's propositions which are omitted
General survey of book:

A false Corollary
A plethora of negatives
A superfluous *datum*
Cumbrous proof of Euc. I. 24
An unintelligible Corollary
A unique 'Theorem of equality'
A bold assumption
Two cases of '*Petitio Principii'*
A problem 3½ pages long >
A fifth case of '*Petitio Principii'*
A sixth

Summing-up, and rejection of Manual.

§2: PIERCE
Treatment of Line

Introduction of Infinitesimals
Treatment of Parallels
Angle viewed as ‘difference of direction’
Assumption of Axiom ‘different Lines may have same direction’
List of Euclid's Theorems which are omitted
Manual *not* adapted for beginners

§3: WILLOCK
Treatment of Parallels
Virtual assumption of Axiom ‘different Lines may have the same direction’ '
Assumption of Axiom ‘separational Lines have the same direction’
General survey of book:

Difficulties introduced too soon
Omission of ‘coincidental’ Lines
‘Principle of double conversion’ discussed, and condemned as illogical
Mysterious passage about ‘incommensurables’

Manual rejected

ACTO III: ***Manuals which adopt Euclid's treatment of Parallels***

ESCENA I

§1: Introductory

§ 2: **CHAUVENET**
General survey

§ 3: LOOMIS
General survey

§ 4: MORELL
Treatment of Line
Treatment of Angle
Treatment of Parallels
General survey:

‘Direct’, ‘reciprocal’, and ‘contrary’ Theorems
Sentient points
A false assertion
A speaking radius
Ratios and common measures
Derivation of ‘homologous’
Mensuration of areas 146
A logical *fiasco*

Manual rejected

§ 5: REYNOLDS
General survey
List of Euclid's Theorems omitted

§ 6: WRIGHT
Quotations from preface
General survey

Specimen of verbose obscurity

ESCENA II

§ 1: *Syllabus of the Association for the Improvement of Geometrical Teaching*
Introduction of *Nostradamus*, a member of the Association
Treatment of Line
Treatment of Angle
Treatment of Parallels
Test for meeting of finite Lines
Re-arrangement of Euclid's Theorems
General survey:

A 'Theorem' is a 'statement of a Theorem'
Rule of Conversion
Miscellaneous inaccuracies

Summing-up

§ 2: *Wilson's 'Syllabus'-Manual*
Introductory
A Theorem is a 'statement of a Theorem'
Rule of Conversion
Every Theorem a 'means of measuring'
'Straight angles'
Miscellaneous inaccuracies
The Manual's one great merit
No test for meeting of finite Lines
Propositions discussed in detail:

An important omission
An illogical conversion
'Un enfant terrible'

Summary of results:

Of 73 propositions of Euclid, this Manual has 14 omitted;
43 done as in Euclid;
10 done by new but objectionable methods, viz.

1 illogical;
1 'hypothetical construction';
2 needlessly using 'superposition';
2 algebraical;
4 omitting the diagonals of Euc. II.;
6 done by new and admissible methods.

No reason for abandoning Euclid's sequence and numeration
Nor for regarding this Manual as anything but a revised Euclid
Summing-up

ACTO IV: [Minos y Euclides] ***Manual de Euclides***

§ 1: *Treatment of Pairs of Lines*
Modern treatment of Parallels
Playfair's Axiom
Test for meeting of finite Lines

§ 2: *Euclid's constructions*
'Arbitrary restrictions'
'Exclusion of hypothetical constructions'

§ 3*: Euclid's demonstrations*
PAGE
'Invariably syllogistic form'
'Too great length of demonstration'
'Too great brevity of demonstration'
'Constant reference to axioms'

§ 4*: Euclid's style*
Artificiality, unsuggestiveness, and want of simplicity

§ 5*: Euclid's treatment of Lines and Angles*

Treatment of' Line
Treatment of Angle :

'declination from' accepted
must be less than sum of two right angles
'multiple angles' in VI. 33
proof for Ax. 10 accepted

§ 6: ***Omissions, alterations, and additions, suggested ly Modern Rivals***
Omission of I. 7 suggested
Reasons for retaining it:

needed to prove I. 8
not included in new I. 8
proves *rigidity* of Triangle .
I. 7, 8 analogous to III. 23, 24
bears on practical science

Omission of II. 8 suggested

Reason for retaining it, its use in Geometrical Conic Sections

Alterations suggested:

New proofs for I. 5
'hypothetical construction'
superposition
treating sides as 'obliques'
treating sides as radii of a Circle
Inversion of order of I. 8, 24; rejected
Inversion of order of I- 18, 19, 20; do.
Fuller proof of I. 24 ; accepted
Algebraical proofs of II. ; rejected

Additions suggested:

New Axiom; accepted

Two new Theorems ; do.

§7: ***The summing-up***
Euclid's farewell speech

APPENDICES

I. Extract from Mr. Todhunter's essay on ‘Elementary Geometry’ included in ‘The Conflict of Studies, &c’

II. Extract from Mr. De Morgan's review of Mr. Wilson's Geometry, in the ‘Athenoeum’ for July 1 8, 1868

III. Proof that, if any one proposition of Table II be granted as an Axiom, the rest can be deduced from it

IV. List of propositions of Euc. I, II, with references to their occurrence in the manuals of his Modern Rivals:

§1. References to Legendre, Cuthbertson, Henrici, Wilson, Pierce and Willock
§2. References to the other Modern Rivals

Nota:

[1] En 2000 Jimbo Wales creó **Nupedia**, un proyecto de enciclopedia libre basado en un ambicioso proceso de revisión por pares. Debido al lento avance del proyecto, en 2001 se creó un wiki –UseModWiki–vinculado a Nupedia cuya finalidad inicial era agilizar la creación de artículos de forma paralela, antes de que éstos pasaran al sistema de revisión por expertos. El éxito de aquel proyecto paralelo – **Wikipedia**– acabó eclipsando a **Nupedia**, que dejó de funcionar en 2003.